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SUBSTITUTION BETWEEN ENERGY AND NON-ENERGY INPUTS IN THE NETHERLANDS 1950–1976*

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1. INTRODUCTION

The oil crisis of 1973 brought home the immediate dependence of economic activity on energy supplies and reinforced the anxieties about the long-term prospects raised by the Meadows [1972] report. Today, the general view that scarcity of energy will affect output or at least impede its growth seems so obvious as to require no argument. The effects of the embargo as a physical restriction were of course readily demonstrated by imposing bottleneck constraints in an input-output model.

It is not so easy, however, to enlarge on a longer-term perspective. This contains the threat of a continued increase in the real costs of energy rather than of sudden disruptions of supply, and it is clear that the response to this will involve substitution. For if the popular view is that energy scarcity reduces economic growth just as cheap, abundant supplies favor it, the underlying belief is that these effects arise precisely because the economic process responds to these stimuli by adaptation.

One should wish for an economic model that corresponds to these simple beliefs and demonstrates inter alia why output should react to variations in factor prices. The present paper falls short of this objective, as it is limited to the substitution between factors in response to price changes without explicit consideration of the level of output. We do, however, include energy among the factors of production, and thus open the way to the introduction of non-factor inputs in the aggregate production function.

The next two sections present the economic model which is based on the Generalized Cobb-Douglas cost function developed by Diewert [1973b]. In Section 4 we discuss the construction of the relevant annual aggregate data for the Dutch economy 1950–1976. Section 5 is devoted to estimation problems. In the last two sections we assess the validity of the model by fitting it to the data, and relate our results to the existing literature.

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2. THE PROBLEM OF VALUE ADDED; HOMOTHETIC SEPARABILITY

Classical economists recognized three primary factors of production, viz., labor, capital, and land or natural resources. The latter was quietly dropped when Douglas introduced the first explicit production function in 1928 and it passed into oblivion until it was recalled by Meadows [1972]. The vast majority of production function studies by economists deal with labor and capital only and refer to value added rather than to output. Material or "non-factor" inputs are omitted from the production function and substracted from its result. This elimination of particular inputs implies certain assumptions about their rôle in production, which we will now investigate.

Our starting point is a positive, nondecreasing, and continuous aggregate production function

$$Y = F(K, L, E, M, A),$$

which relates gross output Y to the services of four aggregate inputs: capital (K), labor (L), energy (E), and all other materials (M). A is a technology index. The formulation of such a function presupposes the existence of consistent aggregate indices for the inputs K, L, E and M.[2] It is, however, by no means certain that such indices exist. Berndt and Christensen [1973], [1974] have investigated the existence of consistent aggregate indices of labor and capital in U.S. manufacturing. They found that equipment and structures could be consistently aggregated, but that this was not possible for production workers and non-production workers.[3] In the present case we use K and L without a further examination of this issue; while a case can be made for disaggregating either variable, we have decided not to follow this route since it would increase sharply the number of parameters, which is undesirable from the point of view of estimation. The reason why we distinguish between M and E will be clarified in the sequel.

In the context of a production function the concept of real value added has an economic interpretation if (i) the prices p_E, p_M, and p_Y move in fixed proportions, or (ii) K and L are homothetically separable from E and M. Condition (i) is discussed in Diewert [1973a] in the context of Hicks' aggregation theorem. It does not apply in the instance we have in mind, since the price of energy fell quite considerably in comparison with the price of produced goods. The second condition holds if and only if we can write:[4]

$$F(K, L, E, M, A) = \bar{F}(g(K, L), E, M, A).$$

The subfunction $g(K, L)$ is homothetic and is identified as real value added.

[2] See Blackorby, Lady, Nissen, and Russell [1970] for a further discussion of consistent aggregation and homothetic separability.

[3] However, their tests place restrictions on the implied aggregates which are too severe. See Blackorby, Primont, and Russell [1977a], and Denny and Fuss [1977].

[4] See Blackorby, Primont, and Russell [1977b, Section 3. 4. 2].

This separability assumption implies severe restrictions on the Allen partial elasticities of substitution (AES) between pairs of inputs.[5] If condition (ii) is satisfied, it turns out that the elasticity of substitution between energy and *any* capital or labor input must be the same. This seems highly implausible, both from intuitive grounds and from the literature. We would expect energy and most capital services to be complements ($AES<0$), and energy and (unskilled) labor to be substitutes ($AES>0$).

We conclude that there is no theoretical justification for the use of real value added as a measure of production, and that we should therefore include E and M in the production function. Yet limitations of data collection prevent us from taking this course. We are able to construct the relevant data for E, but to include M would entail a series of estimates of the annual use in production of raw materials, both imported and home produced; in the latter case the net contribution of land and other natural resources must be assessed. This task is beyond us. Reluctantly we act as though K, L, and E are homothetically separable from M. This enables us to write:

$$Y = F(K, L, E, M, A) = \tilde{F}(H(K, L, E, A), M, A).$$

If, in addition, we assume F twice continuously differentiable and strictly increasing in its arguments (rather than just continuous and nondecreasing), then so is H. Therefore, if H exhibits extended neutral technical change, then in H, A is strictly independent of K, L, and E (and vice versa), i.e.,

$$H(K, L, E, A) = \alpha(A)\cdot\tilde{f}(K, L, E).$$ [6]

In that case, the production function F can be written as

$$Y = \tilde{F}(\alpha(A)\cdot\tilde{f}(K, L, E), M, A).$$

Since $\tilde{f}$ is homothetic, it can be written as $\phi(f)$, where f now is positively linearly homogeneous. Moreover, since $\tilde{F}$ is strictly increasing in $\tilde{f}$, ϕ can be absorbed in $\tilde{F}$:

$$Y = \hat{F}(\alpha(A)\cdot f(K, L, E), M, A).$$

If, finally, we assume $\tilde{f}$ quasi-concave, then f too is quasi-concave. But, since f is positive and positively linearly homogeneous it follows from Newman [1969, Theorem 8] that f is concave. Hence, under the foregoing assumptions on F, H, and $\tilde{f}$, the subfunction f is positive, positively linearly homogeneous, and concave.

In the remainder of this paper we shall be concerned with estimating this function $f(K, L, E)$. As the value of f (denoted as z) cannot be observed, direct estimation of f is impossible. In the next section we will indicate how we can

[5] See Allen [1938] for a definition of AES. For the AES to be wellbehaved we need additional conditions on the cost function which is dual to the production function F. See Blackorby, Primont, and Russell [1977b, Section 7. 1. 2].

[6] Blackorby, Primont, and Russell [1977b, Section 7.2].

get round this difficulty.

3. AN EXTENSION OF DIEWERT'S GENERALIZED COBB-DOUGLAS UNIT COST FUNCTION

In recent years many attempts have been made to estimate production functions indirectly by means of cost-minimizing or profit-maximizing conditions that hold at given input prices. The optimal values of input quantities or of costs can then be expressed as a function of these prices. The oldest example of such an approach is the estimation of Cobb-Douglas production function elasticities from income shares. It has, however, become particularly widespread since Arrow et al. [1961] regressed value added per worker on the wage rate in order to estimate the elasticity of substitution. The use of cost functions, which is equally based upon an assumption about cost minimization for each given output, has been much stimulated by the clarification in Shephard [1953] and the introduction of the translog function by Christensen, Jorgenson and Lau [1971]. Following Diewert [1974] we characterize the duality between cost and production function as follows: Consider an n factor production function f, where $z=f(x_1\cdots x_n)=f(x)$ means that z is the maximal amount of output which can be produced during a given period of time using x_i units of input i $(i=1\cdots n)$. Then, corresponding to f, there is a cost function $C(z, p)$ defining for each $z>0$ and $p\gg 0$ (i.e., $p_i>0$, $i=1\cdots n$) the minimal cost of obtaining z. If f is regular[7] then there is a one-to-one correspondence between cost and production function, with the latter given in terms of the former by

$$f(x) = 1/\max_{p\geqq 0}\{c(p)\,|\,p'x = 1\},$$

where $c(p)\equiv C(1, p)$ (the unit cost function) is also regular.

From this unique correspondence between a regular production function f and a regular unit cost function c it might seem irrelevant whether one chooses f or c to describe the technology. This, however, is not the case for at least two reasons: First, it is usually not possible to derive explicitly f from c or c from f, and secondly, knowledge of c enables us to find the derived demand functions and the cost shares by means of Shephard's lemma.

Thus, let $c(p)$ be a regular and differentiable unit cost function. Applying Shephard's lemma, we obtain the system of derived demand functions as

$$x_i(z, p) = z\,c_i \qquad i = 1,\ldots, n,$$

where $x_i(z, p)$ is the cost minimizing demand for input i needed to produce z and $c_i=\partial c(p)/\partial p_i$.

Clearly,

[7] In this context we shall say that $f(x)$ is regular if for all $x\gg 0$ f is positive, positively homogeneous of degree one, and concave. Note that a regular function is nondecreasing. See Diewert [1974, p. 111].

$$\sum_i p_i x_i = z \sum_i p_i c_i = zc(p),$$

using Euler's theorem on linear homogeneous functions. Eliminating the (unknown) output z we find expressions for the cost shares y_i:

$$y_i \equiv \frac{p_i x_i}{\sum_i p_i x_i} = \frac{z p_i c_i}{z c(p)} = \frac{p_i c_i}{c} \qquad i = 1, \ldots, n.$$

For purposes of estimation we must employ a specific functional form for c. We have searched for a highly general functional form, one that places no a priori restrictions on the Allen partial elasticities of substitution (*AES*), and one that can be interpreted as a second order approximation to an arbitrary twice-differentiable unit cost function. Several functional forms satisfy these requirements: the translog function (Christensen, Jorgenson and Lau [1971], [1973]), the Generalized Leontief (Diewert [1971]) and the Generalized Cobb-Douglas (Diewert [1973b]). Differences among these 'flexible' functional forms have been analyzed by Berndt, Darrough, and Diewert [1977]. On theoretical and econometric grounds they find the translog functional form preferable, although, when symmetry restrictions were imposed, results obtained from the three functional forms were basically similar.

We opted for an extension of Diewert's Generalized Cobb-Douglas (GCD) unit cost function by defining the following n factor unit cost function

$$(1) \qquad c(p) = \Theta \prod_{i=1}^{n} \prod_{j=1}^{n} \psi_{ij}{}^{\beta_{ij}},$$

where

$$\psi_{ij} = \alpha_i p_i + \alpha_j p_j,\ \alpha_i > 0,\ \beta_{ij} = \beta_{ji},\ \sum_{ij} \beta_{ij} = 1,\ \Theta > 0 \qquad i, j = 1, \ldots, n.$$

It is easy to see that $c(p)$ as defined in (1) is positive and (positively) linear homogeneous of degree one in p. Further, if $\beta_{ij} > 0$ $(i, j = 1 \cdots n)$ it is also concave.[8] If some of the β_{ij} are negative, then we will have to check the fitted unit cost function for concavity at each observation.

Logarithmically differentiating we find

$$\frac{\partial \log c(p)}{\partial p_i} = 2\alpha_i \sum_k \frac{\beta_{ik}}{\psi_{ik}}$$

and

$$\frac{\partial^2 \log c(p)}{\partial p_i \partial p_j} = -2\alpha_i \alpha_j \left(\frac{\beta_{ij}}{\psi_{ij}^2} + \delta_{ij} \sum_k \frac{\beta_{ik}}{\psi_{ik}^2} \right),$$

where δ_{ij} is the Kronecker delta: $\delta_{ij} = 0$ if $i \neq j$, $\delta_{ii} = 1$. The cost share equations are

$$(2) \qquad y_i = p_i \frac{\partial \log c(p)}{\partial p_i} = \sum_k \left(\frac{2\alpha_i p_i}{\psi_{ik}} \right) \beta_{ik} \qquad i = 1, \ldots, n.$$

[8] See Diewert [1973b, p. 10].

The unit cost function (1) reduces to Diewert's GCD function by putting $\alpha_i = \frac{1}{2}$ $(i=1\cdots n)$. Diewert's form has the advantage that the share equations (2) become linear in the unknown parameters. However, estimates of parameters and substitution elasticities and results of tests of hypotheses lack invariance to arbitrary multiplicative scaling of the input prices. This property renders Diewert's GCD form unattractive for empirical work.[9] The share equations (2), resulting from the extended GCD form, do not suffer from this drawback.

We shall estimate the parameters of (2) by applying these equations to time series of aggregate annual data on cost shares and factor prices for the Netherlands. Many assumptions are involved in the passage from the theoretical optimum conditions that are reflected in (2) to their direct empirical verification by means of annual aggregates, and several of these are easily challenged. Thus the estimation procedure treats factor prices as predetermined variables, even though we are dealing with aggregates, and implies instantaneous adjustment in respect to all factors of production. Equally strong assumptions are involved in the construction of the capital data, as we shall note below. As matters stand we have not yet been able to remedy these shortcomings in our analysis. We shall again draw attention to their presence when we discuss our results.

Before we turn to the construction of data on cost shares and price indices in the next section, we will relate the parameters α_i and β_{ij} to the *AES* σ_{ij} between inputs i and j. Uzawa [1962, p. 293] has shown that for a linear homogeneous unit cost function $c(p)$ we have

$$\sigma_{ij} = 1 + \frac{\partial^2 \log c(p)/\partial p_i \partial p_j}{(\partial \log c(p)/\partial p_i)(\partial \log c(p)/\partial p_j)}.$$

The σ_{ij} of the extended GCD function can then be expressed in the parameters β_{ij} and α_i as follows:

$$(3) \qquad \sigma_{ij} = 1 - \frac{1}{2} \frac{\dfrac{\beta_{ij}}{\psi_{ij}^2} + \delta_{ij} \sum_k \dfrac{\beta_{ik}}{\psi_{ik}^2}}{\left(\sum_k \dfrac{\beta_{ik}}{\psi_{ik}}\right)\left(\sum_k \dfrac{\beta_{jk}}{\psi_{jk}}\right)}.$$

We finally note that[10]

$$\sum_i p_i c_{ij} = \sum_i y_i \sigma_{ij} = 0, \qquad \text{for all } j,$$

which implies that the Hessian matrix of the unit cost function (c_{ij}) and the substitution matrix (σ_{ij}) are singular.

[9] I am grateful to the referee for drawing my attention to this point. The lack of invariance of the GCD form was first noted by Wales and Woodland [1977, p. 116].

[10] Differentiate both sides of $c(p) = \sum_i p_i c_i$ with respect to p_j. This gives $\sum_i p_i c_{ij} = 0$. Further, $\sum_i y_i \sigma_{ij} = \sum_i \left(\frac{p_i c_i}{c}\right)\left(\frac{c c_{ij}}{c_i c_j}\right) = \frac{1}{c_j} \sum_i p_i c_{ij} = 0.$

4. THE DATA

The data consists of annual time-series for the Netherlands, 1950–1976. As to labor and capital, they refer to the enterprises sector as defined in the National Accounts, exclusive of the production of crude oil, natural gas and coal. This ensures that no energy is generated within the aggregate thus defined, although it is of course transformed from one form into another, as in electricity generation. All primary energy can therefore properly be treated as an input.

We require data on cost shares and prices of labor, energy and capital. The cost shares are easily obtained once we have a volume or quantity series and a price series that refers to the price per unit of measurement of volume or quantity. We shall note these as we go along.

We have freely drawn on official statistics and on studies of the Central Planning Bureau, the Ministry of Economic Affairs and the Ministry of Finance. The labor data were readily available, the energy figures required some adjustment and the capital data are based on extensive theoretical considerations.

We shall briefly indicate the main points of each series below; a full documentation (in Dutch) is available from the author on request.

For *labor* the quantity is defined as the total labor force employed in the enterprises sector, mining excluded, corrected for the length of the working year and the incidence of sick leave. Labor input thus corresponds to the number of man-hours worked. The price is derived from average wage costs per employee (including social security contributions), reduced to the price in guilders of a man-hour for the enterprises sector as a whole.

For *energy* the input of the enterprises sector is obtained by reducing the known total consumption of primary energy by the estimated gross consumption of private households. This includes the consumption of coal, oil and gas for domestic use (mainly heating), the gross energy counterpart of domestic electricity demand and private petrol consumption. For the price of energy we used an existing index of the average purchase price of primary energy by industry, which was converted to a price in guilders per Kcal.

The method of measuring real *capital* input and capital service prices is based on Christensen and Jorgenson [1969]. This calls for the construction of rather delicate indices, and we shall explain the procedure at some length.

To begin with we construct a series for the volume of capital stock, again excluding the mining sector. We distinguish among three types of capital goods, viz., equipment, transport equipment (vehicles and ships) and buildings. For each type the capital stock is constructed from past investment in 1963 guilders as

$$K_i(t) = I_i(t) + (1 - \mu_i)K_i(t-1) \qquad i = 1, 2, 3, \tag{4}$$

where $K_i(t)$ capital stock volume at end of year t;
$I_i(t)$ volume of investment during year t;

μ_i depreciation rate.[11]

In constructing the price of capital service input for each type, we assume for the moment that the investment price of an asset equals the present value of its future services, evaluated at the price we wish to ascertain. This presupposes perfect foresight on the part of the firm. Also it is assumed that the service flow from a given asset declines geometrically over time. Disregarding taxes on the capital service yield, we can write the equality at issue as

$$(5) \qquad q(t) = \sum_{j=t}^{\infty}\left[(1-\mu)^{j-t}p(j+1)\prod_{s=t+1}^{j+1}\frac{1}{1+r(s)}\right],$$

where

$q(t)$ price index of investment, 1963 = 1;
$p(t)$ capital service price;
$r(t)$ discount rate
all at year t.

From (5) follows the well-known expression

$$(6) \qquad p(t) = q(t-1)r(t) + q(t)\mu - (q(t) - q(t-1)).$$

Allowing now for taxes that are levied on the capital services value as it is obtained by a firm, we arrive at the formula employed by Christensen and Jorgenson [1969, p. 304], adapted to the Dutch tax system:

$$(7) \quad \pi(t) = \frac{1 - a(t) - u(t)B(t)}{1 - u(t)}(r(t)q(t-1) + \mu q(t) - (q(t) - q(t-1))),$$

where $a(t)$ is investment tax credit, $B(t)$ is the discounted value of depreciation charges stemming from a current guilder of capital expenditures, and $u(t)$ is the effective corporate profits tax rate. The data employed for the calculation of $K_i(t)$ and $\pi_i(t)$ is presented in Table 1. As benchmarks for the capital stock in 1949 we used $K_1 = 15.7$, $K_2 = 8$, and $K_3 = 15.6$ (billions of 1963 guilders). In Table 2 we give the resulting values of π after aggregation of the three types of capital goods.[12] It is hard to believe that the instant adjustment to price changes implied by our model would in effect take place with such fluctuations in capital price. Since the fluctuations in π are mainly caused by the term $q(t) - q(t-1)$, i.e., by the capital gains, we concluded that these capital gains should play a less dominant rôle in the determination of the price of capital services. Thus we assumed that the investors ignore the anticipated capital gain when they consider purchasing a capital good, in which case the term $q(t) - q(t-1)$ can be eliminated from equation (7). This gives

$$(8) \qquad \pi^*(t) = \frac{1 - a(t) - u(t)B(t)}{1 - u(t)}(r(t)q(t-1) + \mu q(t)).$$

[11] For μ_i we took μ_1 (equipment) = .06; μ_2 (transport) = .10 and μ_3 (buildings) = .03.

[12] We used the unit cost function (1) as an aggregator function. It can be viewed as a second-order approximation to an arbitrary well-behaved aggregator function.

In Table 2 we present the price indices and the corresponding input costs we have searched for.

5. THE ESTIMATION PROCEDURE

Having collected data on cost shares and input prices during T years, we may now ask how to estimate the parameters α_i and β_{ij} from the system of equations (2), if we suppose the prices to be exogeneous.

Imposing the symmetry constraints $\beta_{ij}=\beta_{ji}$ and adding a stochastic disturbance vector, we obtain

$$(9) \qquad \begin{pmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \phi_{12} & \phi_{13} & 0 \\ 0 & 1 & 0 & \phi_{21} & 0 & \phi_{23} \\ 0 & 0 & 1 & 0 & \phi_{31} & \phi_{32} \end{pmatrix}_t \begin{pmatrix} \beta_{11} \\ \beta_{22} \\ \beta_{33} \\ \beta_{12} \\ \beta_{13} \\ \beta_{23} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix} \qquad t = 1,\ldots, T,$$

where $\phi_{ij}=2\alpha_i p_i/\psi_{ij}$. For short:

$$y_t^* = X_t^*\beta^* + \varepsilon_t^* \qquad t = 1,\ldots, T.$$

The following two characteristics of the model should be noted:

$$(10) \qquad s'y_t^* = 1 \quad \text{with} \quad s' = (1\ 1\ 1)$$

$$(11) \qquad q'\beta^* = 1 \quad \text{with} \quad q' = (1\ 1\ 1\ 2\ 2\ 2),$$

where (10) stems from the fact that the y_i are shares and (11) is the linear homogeneity condition.

Now suppose $E\varepsilon^*=0$ and $E\varepsilon^*\varepsilon^{*\prime}=\Omega^*$, where $\varepsilon^{*\prime} = (\varepsilon_1^{*\prime}, \varepsilon_2^{*\prime},\ldots, \varepsilon_T^{*\prime})$. Then

$$1 = s'y_t^* = s'X_t^*\beta^* + s'\varepsilon_t^*.$$

Also

$$0 = Es'\varepsilon_t^* = 1 - s'X_t^*\beta^*.$$

Therefore $s'X_t^*\beta^*=1$ and $s'\varepsilon_t^*=0$, so that Ω^* must be singular.

Notice that $s'X_t^*=q'$, so that conditions (10) and (11) imply that one of the equations in (9), say the third, becomes superfluous. We therefore delete the third equation and write:

TABLE 1

TIME-SERIES FOR r^*, u, I_i, q_i, B_i^{**} AND a_i

	r	u	Volume of investment			Price index of investment			Discounted value of depreciation charges stemming from a current guilder of capital expenditures			Investment tax credit		
			I_1	I_2	I_3	q_1	q_2	q_3	B_1	B_2	B_3	a_1	a_2	a_3
1949	.0833					.6526	.7695	.4915	.6585	.6117	.3913	—	—	—
1950	.1313	.40	1901	653	1407	.6782	.7929	.5176	.7328	.6902	.5276	—	—	—
1951	.1684	.50	1889	543	1303	.7599	.8767	.5955	.8181	.7688	.5849	—	—	—
1952	.0300	.50	1675	541	1333	.8549	.8890	.6442	.8022	.7524	.5698	—	—	—
1953	.0300	.46	1870	681	1678	.8400	.8844	.6199	.7794	.7297	.5487	.1257	.1257	.1257
1954	.0707	.46	2179	995	1677	.8277	.9003	.6427	.7875	.7432	.5505	.1700	.1700	.1700
1955	.0561	.43	2486	1375	1593	.8414	.9270	.6962	.7838	.7455	.5438	.1699	.1699	.1699
1956	.0427	.43	2946	1578	1946	.8859	.9468	.7710	.7604	.7222	.5206	.1425	.1508	.1425
1957	.0802	.47	2880	1903	2142	.9348	.9235	.8397	.7743	.7338	.5269	—	.0430	—
1958	.0659	.47	2326	1483	2018	.9471	.9393	.8626	.7801	.7358	.5272	.1057	.1218	.0870
1959	.0300	.47	2615	1651	2142	.9345	.9704	.8412	.7533	.7185	.5155	.1473	.1473	.1473
1960	.0647	.47	3059	2113	1693	.9445	.9488	.8763	.7476	.7144	.4926	.1118	.1211	.1118
1961	.0412	.47	3610	2001	1801	.9485	.9665	.9068	.7210	.6869	.4630	.0909	.1054	.0909
1962	.0632	.47	3779	2153	1848	.9534	.9686	.9375	.7087	.6759	.4529	.0889	.1031	.0889
1963	.0729	.45	3873	2010	1785	1.0000	1.0000	1.0000	.7009	.6691	.4054	.0878	.0913	.0878
1964	.1021	.45	4275	1893	2310	1.0627	1.0436	1.0904	.6788	.6610	.3257	.0903	.0939	.0076
1965	.0685	.45	4421	2114	2308	1.1019	1.0454	1.1583	.6593	.6516	.3159	.0897	.0771	—
1966	.0764	.47	5011	2157	2645	1.1458	1.0529	1.2359	.6472	.6409	.3079	.0910	.0782	—
1967	.0744	.47	5157	2191	3000	1.1322	1.1032	1.2812	.6313	.6258	.3154	.0910	.0782	.0327
1968	.0470	.46	5718	2569	3293	1.1151	1.0987	1.3205	.5944	.5897	.3593	.0875	.0753	.0875
1969	.1054	.46	5857	2536	3250	1.1549	1.0913	1.4052	.5900	.5964	.3586	.0167	.0302	.0167
1970	.0690	.46	7205	2855	3524	1.2334	1.1543	1.5266	.5622	.5725	.3425	—	.0188	—
1971	.1127	.47	6843	3167	3435	1.3393	1.2240	1.6702	.5566	.5668	.3396	—	.0186	—
1972	.1135	.48	6333	2813	3052	1.3765	1.2892	1.8402	.5507	.5713	.3359	—	.0240	—
1973	.1266	.48	6924	3524	3161	1.3698	1.3245	1.9773	.5513	.5844	.3359	—	.0277	—
1974	.1301	.48	7463	3366	2946	1.5303	1.4395	2.2304	.5542	.5915	.3391	.0519	.0612	.1305
1975	.1319	.47	6708	3260	2680	1.7074	1.6284	2.4538	.5587	.5963	.3696	.0674	.0617	.1937
1976	.1227	.48	6673	2863	2728	1.8154	1.7449	2.6739	.5587	.5963	.3783	.1010	.0519	.1684

* The discount rate r was calculated from $1+r(t)=(1+\theta)(1+\omega(t))$, where θ is time preference (constant over time at $\theta=.03$), and $\omega(t)$ is the inflation rate.

** For the calculation of B we used the straight-line depreciation formula, taking into account accelerated depreciation allowances.

TABLE 2

PRICE INDICES AND INPUT COSTS OF ENERGY, LABOR AND CAPITAL. DUTCH ENTERPRISES 1950–1976*

	Price indices (1963 = 1.0)				Input costs (millions of current guilders)			
	P_E	P_L	π	π^*	E	L	K	K^*
1950	.7767	.3736	1.3269	.9936	753	9511	4638	5930
1951	.9689	.4224	.9873	1.2890	985	10786	3573	7965
1952	1.0758	.4389	.0523	.5728	1079	11105	195	3641
1953	1.0475	.4572	1.0060	.4620	1078	11714	3904	3061
1954	1.0690	.4946	.9589	.6179	1192	12955	3917	4310
1955	1.1326	.5424	.5689	.5556	1347	14442	2464	4108
1956	1.1947	.5909	.2945	.5510	1548	15934	1369	4373
1957	1.3554	.6607	1.0289	1.0161	1568	17789	5131	8651
1958	1.2856	.6854	1.1505	.8095	1637	18350	5996	7203
1959	1.1390	.6959	1.0346	.5375	1529	18916	5663	5023
1960	1.0321	.7570	1.1819	.8163	1533	20888	6808	8028
1961	.9658	.8369	.9565	.7062	1509	22748	5811	7326
1962	.9696	.9072	1.3074	.8939	1657	24600	8371	9772
1963	1.0000	1.0000	1.0000	1.0000	1859	27207	6701	11441
1964	.9386	1.1470	1.2676	1.3770	1865	31887	8936	16574
1965	.8954	1.2803	1.2817	1.1642	1922	35759	9499	14731
1966	.9558	1.4233	1.4230	1.3247	2149	39971	11148	17719
1967	.9602	1.5489	1.9322	1.3360	2280	43324	15998	18886
1968	.9587	1.7206	1.6354	1.0299	2575	47668	14403	15485
1969	.9046	1.9878	2.2744	1.8180	2733	54962	21174	28897
1970	1.0571	2.2987	1.0366	1.5298	3547	63000	10315	25990
1971	1.2121	2.6303	1.8844	2.2603	4332	71260	19868	40688
1972	1.1018	2.9948	2.5865	2.4750	4499	79259	28441	46464
1973	1.2664	3.5310	3.8025	2.8005	5462	91345	43891	55192
1974	1.6402	4.1267	1.7070	2.6989	7006	104763	20605	55622
1975	2.2681	4.7944	2.0206	2.8395	9403	118510	25199	60462
1976	2.9834	5.3086	2.9189	2.9785	13780	129684	37406	65171

* As to the input costs for capital, K corresponds to the price index π, and K^* to π^*.

$$(12)\qquad \begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \phi_{12} & \phi_{13} & 0 \\ 0 & 1 & \phi_{21} & 0 & \phi_{23} \end{pmatrix}_t \begin{pmatrix} \beta_{11} \\ \beta_{22} \\ \beta_{12} \\ \beta_{13} \\ \beta_{23} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \qquad t = 1,\ldots, T.$$

In the above system y_{3t} and β_{33} do not appear, so that both constraints have been resolved. We abbreviate (12) to:

$$(13)\qquad y_t = X_t\beta + \varepsilon_t \qquad t = 1,\ldots, T.$$

Combining annual observations we may write

$$y = X\beta + \varepsilon,$$

where $y'=(y_1', y_2',\ldots, y_T')$; $X'=(X_1', X_2',\ldots, X_T')$; $\varepsilon'=(\varepsilon_1', \varepsilon_2',\ldots, \varepsilon_T')$.

On closer inspection of (12) we see that the two equations are related, since the α's and β_{12} appear in both. More important, the two equations may be disturbance-related, that is, the neglected variables in the two equations may be the same (or at least correlated). Certain *a priori* restrictions as to the covariance structure are however needed: we assume that the disturbances are distributed independently over successive years and that the covariance matrix is the same in each year. Let $E\varepsilon\varepsilon'=\Omega$, then Ω is a block-diagonal matrix, that is $\Omega=I_T\otimes\Gamma$, where $\otimes$ denotes the Kronecker product operator and Γ is a symmetric positive definite 2×2 matrix

$$\Gamma = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{12} & \gamma_{22} \end{pmatrix}. \tag{14}$$

Assuming normal disturbances we shall estimate the α's, β and Γ by the method of maximum likelihood (ML). Since the ε_t are identically and independently distributed as $N(0, \Gamma)$, the likelihood of the sample $(\varepsilon_1, \varepsilon_2,\ldots, \varepsilon_T)$ is

$$\prod_t \left[(2\pi)^{-1}|\Gamma|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\varepsilon_t'\Gamma^{-1}\varepsilon_t \right) \right].$$

Of course, the loglikelihood is

$$\Lambda = -T\log 2\pi + \frac{1}{2}T\log|\Gamma^{-1}| - \frac{1}{2}\sum_t \varepsilon_t'\Gamma^{-1}\varepsilon_t.$$

From the likelihood we may derive the ML equations and the information matrix[13]. Denote

$$\theta' = (\gamma_{11}, \gamma_{12}, \gamma_{22}),$$

then the ML equations are

$$\begin{array}{llll} & \text{(i)} & \hat\beta = (\sum_t \hat X_t'\hat\Gamma^{-1}\hat X_t)^{-1}(\sum_t \hat X_t'\hat\Gamma^{-1}y_t), & \\ (15) & \text{(ii)} & (\sum_t e_t'\hat\Gamma^{-1}\hat\Xi_t^i)\hat\beta = 0 & i = 1, 2, 3, \\ & \text{(iii)} & T\operatorname{tr}(\hat C_h\hat\Gamma) = \sum_t e_t'\hat C_h e_t & h = 1, 2, 3, \end{array}$$

where

$$C_h = \frac{\partial\Gamma^{-1}}{\partial\theta_h}, \quad \Xi_t^i = \frac{\partial X_t}{\partial\alpha_i}, \quad \text{and} \quad e_t = y_t - \hat X_t\hat\beta.$$

The third ML equation can be developed a little further. Since

[13] See Magnus [1978] for a complete derivation in a linear context.

$$-d\Gamma^{-1} = \Gamma^{-1}(d\Gamma)\Gamma^{-1},$$

we have

$$-\frac{\partial \Gamma^{-1}}{\partial \gamma_{ij}} = \Gamma^{-1}\frac{\partial \Gamma}{\partial \gamma_{ij}}\Gamma^{-1} = \Gamma^{-1}Y^{ij}\Gamma^{-1},$$

where Y^{ij} is a (2, 2) matrix with unity in the ij-th and ji-th position and zeroes elsewhere. Further,

$$-\frac{\partial \Gamma^{-1}}{\partial \gamma_{ij}}\Gamma = \Gamma^{-1}Y^{ij}.$$

This leads to

$$-\sum_t e_t'\frac{\partial \Gamma^{-1}}{\partial \gamma_{ij}}e_t = \sum_t e_t'\Gamma^{-1}Y^{ij}\Gamma^{-1}e_t = \mathrm{tr}[\Gamma^{-1}(\sum_t e_t e_t')\Gamma^{-1}Y^{ij}].$$

The equation (15.iii) thus reads

$$T\,\mathrm{tr}[\Gamma^{-1}Y^{ij}] = \mathrm{tr}[\Gamma^{-1}(\sum_t e_t e_t')\Gamma^{-1}Y^{ij}] \qquad i, j = 1, 2,$$

which is equivalent to

$$T\Gamma^{-1} = \Gamma^{-1}(\sum_t e_t e_t')\Gamma^{-1},$$

and thus

$$\hat{\Gamma} = \frac{1}{T}\sum_t e_t e_t'. \tag{16}$$

The ML estimates of the α's, β and Γ are then found as follows: Assume for the moment that the α's are fixed. Then, conditional upon these α's, we can find the maximum of the likelihood function and the estimates of β and Γ by applying an iterative scheme.[14] This can be done for each combination of the α's. The pattern search method was used to select those α's that maximize the likelihood.[15]

We now turn to the derivation of the asymptotic covariance matrices of β, α and θ. The information matrix takes the form

$$\Psi = \begin{bmatrix} \sum_t X_t'\Gamma^{-1}X_t & \Psi_{\alpha\beta} & 0 \\ \Psi_{\alpha\beta}' & \Psi_{\alpha\alpha} & 0 \\ 0 & 0 & \frac{1}{2}\Psi_\theta \end{bmatrix},$$

[14] The scheme is as follows: choose some starting values of $\gamma_{11}, \gamma_{12}, \gamma_{22}$, such that Γ is positive definite, calculate $\hat{\beta}$ from (15.i), the residuals, and compose the new Γ by (16). Then calculate $\hat{\beta}$ again, etc. This iterative scheme is usually called 'iterative Zellner'. It should be noted, however, that Zellner's covariance matrix is $\Gamma \otimes I$, whereas we use $I \otimes \Gamma$.

[15] Note that the search is two rather than three dimensional, since the ϕ_{ij} are invariant to arbitrary scaling of the α's. We therefore put one restriction on the α's, viz., $\sum_i \alpha_i = 1$.

where

$$(\Psi_{\alpha\beta})_{\cdot i} = (\sum_t X_t'\Gamma^{-1}\Xi_t^i)\beta \qquad i = 1, 2,$$

$$(\Psi_{\alpha\alpha})_{ij} = \beta'[\sum_t (\Xi_t^i)'\Gamma^{-1}\Xi_t^j]\beta \qquad i, j = 1, 2,$$

$$(\Psi_\theta)_{ij} = T\operatorname{tr}(C_i\Gamma C_j\Gamma) \qquad i, j = 1, 2, 3.$$

It may be verified that in the present case

$$\Psi_\theta = \frac{T}{[\gamma_{11}\gamma_{22} - \gamma_{12}^2]^2}\begin{pmatrix} \gamma_{22}^2 & -2\gamma_{12}\gamma_{22} & \gamma_{12}^2 \\ \cdot & 2(\gamma_{11}\gamma_{22} + \gamma_{12}^2) & -2\gamma_{11}\gamma_{12} \\ \cdot & \cdot & \gamma_{11}^2 \end{pmatrix}.$$

Thus, the asymptotic covariance matrices are

$$\text{(17)} \qquad \operatorname{cov}(\hat\beta) = [(\sum_t X_t'\Gamma^{-1}X_t) - \Psi_{\alpha\beta}\Psi_{\alpha\alpha}^{-1}\Psi_{\alpha\beta}']^{-1},$$

$$\text{(18)} \qquad \operatorname{cov}\begin{pmatrix} \hat\alpha_1 \\ \hat\alpha_2 \end{pmatrix} = \Psi_{\alpha\alpha}^{-1} + \Psi_{\alpha\alpha}^{-1}\Psi_{\alpha\beta}' \operatorname{cov}(\hat\beta)\Psi_{\alpha\beta}\Psi_{\alpha\alpha}^{-1},$$

$$\text{(19)} \qquad \operatorname{cov}\begin{pmatrix} \hat\gamma_{11} \\ \hat\gamma_{12} \\ \hat\gamma_{22} \end{pmatrix} = 2\Psi_\theta^{-1} = \frac{2}{T}\begin{bmatrix} \gamma_{11}^2 & \gamma_{11}\gamma_{12} & \gamma_{12}^2 \\ \cdot & \frac{1}{2}(\gamma_{11}\gamma_{22} + \gamma_{12}^2) & \gamma_{12}\gamma_{22} \\ \cdot & \cdot & \gamma_{22}^2 \end{bmatrix}.$$

Two peculiarities in (19) are worth noting:

(i) The asymptotic t-ratios are

$$t(\hat\gamma_{11}) = t(\hat\gamma_{22}) = \sqrt{\tfrac{1}{2}T}$$

$$t(\hat\gamma_{12}) = \gamma_{12}\sqrt{T}(\gamma_{11}\gamma_{22} + \gamma_{12}^2)^{-\frac{1}{2}}$$

We see that $t(\hat\gamma_{11})$ and $t(\hat\gamma_{22})$ depend only upon the number of years T.[16]

(ii) The asymptotic correlation coefficients are

$$\rho(\gamma_{11}, \gamma_{12}) = \rho(\gamma_{12}, \gamma_{22}) = \gamma_{12}\sqrt{2}(\gamma_{11}\gamma_{22} + \gamma_{12}^2)^{-\frac{1}{2}}$$

$$\rho(\gamma_{11}, \gamma_{22}) = \gamma_{12}^2/(\gamma_{11}\gamma_{22}).$$

We finally wish to compose the covariance matrix of the total β^* vector. This

[16] This should not be too surprising. In the classical linear regression model $y = X\beta + \varepsilon$, where $\varepsilon = N(0, \sigma^2 I)$, the ML estimator for σ^2 is $\hat\sigma^2 = (1/n)e'e$ (e being the residual vactor), and $\sqrt{n}(\hat\sigma^2 - \sigma^2)$ is asymptotically distributed as $N(0, 2\sigma^4)$. Hence, the asymptotic t-ratio of $\hat\sigma^2$ is $\sqrt{\frac{1}{2}n}$, which depends on the number of observations n only. The model in the text, being a generalization of the classical regression model, possesses this same property.

may be done in the following way:

Define

$$A = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ -1 & -1 & -2 & -2 & -2 \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}$$

where all undesignated elements are zero, then $\operatorname{Cov} \beta^* = \operatorname{Cov} A\beta = A(\operatorname{Cov} \beta)A'$.

6. EMPIRICAL RESULTS

Two models based on the extended GCD unit cost function (1) were estimated, the difference between them lying in the measurement of the price of capital services. In the first model investors ignore the anticipated capital gain when they consider to purchase a capital good. This leads to π^* as defined in (8). The second model uses π (formula (7)) as the price of capital services. Although it is hard to imagine that the fluctuations in π would in effect take place, the second model was estimated as a means of comparison.

To carry out the iterative procedure as described in the previous section, we deleted the capital equation[17] in the passage from (9) to (12), and found a unique convergence point, independent of starting values. The parameter estimates of the two models GCD. 1 (based on π^*) and GCD. 2 (based on π) are (asymptotic t-values in parentheses):

Parameter[18]	GCD. 1		GCD. 2	
α_E	.3515	(3.9368)	.4201	(5.3125)
α_L	.0606	(.6353)	.3289	(3.1104)
α_K	.5879	(7.8303)	.2510	(6.8801)
β_{EE}	.0268	(.3874)	−.0426	(−3.9607)
β_{LL}	.6507	(25.1753)	.6210	(13.4619)
β_{KK}	−.0501	(−.1293)	.0522	(3.0980)
β_{EL}	−.0304	(−.8548)	.0045	(.5768)
β_{EK}	.0959	(11.4015)	.0769	(9.8004)
β_{LK}	.1208	(.5991)	.1033	(4.6663)

More revealing than the parameter estimates are the estimated Allen partial elasticities of substitution (σ_{ij}) as formulated in (3). Allen [1938] showed that the price elasticities of factor demand are related to the σ_{ij} as:

$$\tau_{ij} \equiv \partial \ln x_i / \partial \ln p_j = y_j \sigma_{ij},$$

[17] The results are independent from the equation deleted, as of course they should be.

[18] We shall use E, L and K instead of 1, 2 and 3 for easy reference.

where y_j is the cost share of the j-th input. In Table 3 we present for both models the estimated σ_{ij} and τ_{ij} for ten selected years. These estimates are the essence of this paper. We will make a few comments:

(i) Energy and labor appear to be close substitutes. In the first model σ_{EL} fluctuates around 1.29, and in the second around .95. No doubt energy and certain types of maintenance workers are complements, but the more powerful effect is that energy is a substitute for (unskilled) labor.

(ii) As expected, we find that energy and capital are highly complementary (σ_{EK} is about -2.19 in the first model, and -2.45 in the second).

(iii) We find a slight substitution between labor and capital (σ_{LK} is about .88 and .69 in the two models), which is somewhat lower than in the traditional two-input (capital-labor) studies. However, Parks [1971], applying a five-input model to Swedish manufacturing, finds $\sigma_{LK}=.12$.

(iv) The estimates of the 'own' price elasticities of factor demand τ_{ii} ($i=E$, L, K) show that capital is quite responsive to a change in its own price (τ_{KK} is about $-.48$ and $-.39$ in the two models). Labor appears less responsive (τ_{LL} is about $-.31$ in the first model and $-.20$ in the second). As to energy, τ_{EE} is about $-.23$ and $-.19$ in the two models, but in the second model τ_{EE} is positive from 1967–73 (1970 excepted).

TABLE 3

PRICE ELASTICITIES OF FACTOR DEMAND (τ_{ij}) AND ALLEN PARTIAL ELASTICITIES OF SUBSTITUTION (σ_{ij}), ESTIMATED FROM MODELS GCD. 1 AND GCD. 2. DUTCH ENTERPRISES 1950–1976

		τ_{EE}	τ_{LL}	τ_{KK}	τ_{EL}	τ_{LE}	τ_{EK}	τ_{KE}	τ_{LK}	τ_{KL}	σ_{EL}	σ_{EK}	σ_{LK}
1950	1	.20	−.34	−.53	.79	.03	−.99	−.10	.30	.63	1.21	−3.18	.96
	2	.05	−.26	−.44	.63	.04	−.68	−.10	.22	.53	.93	−2.42	.79
1953	1	−.39	−.31	−.48	.72	.08	−.33	−.11	.22	.59	1.07	−1.32	.88
	2	−.33	−.23	−.41	.67	.06	−.34	−.09	.17	.50	.96	−1.42	.71
1956	1	−.38	−.30	−.47	.73	.08	−.34	−.11	.22	.58	1.08	−1.37	.87
	2	−.71	−.17	−.42	.75	.10	−.05	−.04	.08	.45	.98	−.36	.59
1959	1	−.39	−.30	−.46	.74	.08	−.35	−.11	.21	.57	1.10	−1.42	.85
	2	−.35	−.21	−.38	.69	.06	−.34	−.10	.14	.48	.96	−1.57	.67
1962	1	−.18	−.31	−.48	.81	.05	−.63	−.11	.25	.59	1.22	−2.21	.89
	2	−.11	−.20	−.38	.68	.05	−.57	−.12	.16	.50	.94	−2.48	.69
1965	1	−.14	−.32	−.49	.91	.04	−.78	−.10	.26	.59	1.38	−2.60	.89
	2	−.05	−.19	−.37	.69	.04	−.65	−.14	.14	.50	.93	−3.04	.68
1968	1	−.28	−.30	−.45	.90	.05	−.62	−.11	.23	.56	1.33	−2.22	.84
	2	.13	−.18	−.37	.69	.04	−.81	−.14	.15	.51	.92	−3.76	.69
1971	1	−.18	−.32	−.50	1.07	.03	−.90	−.08	.28	.59	1.62	−2.89	.89
	2	.04	−.17	−.37	.71	.04	−.75	−.15	.14	.52	.93	−3.77	.69
1974	1	−.27	−.31	−.48	1.03	.04	−.75	−.09	.26	.57	1.54	−2.52	.86
	2	−.25	−.17	−.38	.75	.06	−.50	−.17	.11	.55	.95	−3.10	.70
1976	1	−.30	−.30	−.45	.88	.05	−.59	−.11	.23	.56	1.31	−2.14	.83
	2	−.29	−.17	−.37	.73	.06	−.45	−.15	.11	.52	.95	−2.62	.67

(v) For the cost model and the resulting factor demand equations to be consistent with cost minimization, the substitution matrix (σ_{ij}) must be negative semidefinite in each year.[19]

For the first model, this is the case in 21 of the observed 27 years; for the second model only in 16 years. Although such deviations from second-order theoretical requirements are quite common, this is nevertheless a disturbing feature.

(vi) Also, we conclude, somewhat surprisingly, that the results from the two models are quite comparable.

Finally, we shall relate the models GCD. 1 and GCD. 2 to their translog counterparts TL. 1 and TL. 2, and to the existing literature. Berndt and Wood [1975], Hudson and Jorgenson [1974], Fuss [1977], and Griffin and Gregory [1976] all test a translog cost function on manufacturing data with energy as a separate factor input. Berndt-Wood and Hudson-Jorgenson apply U.S. time-series data (1947–71) to a four-inputs (K, L, E and M) translog cost function. Fuss obtains his data sample by pooling time-series (1961–71) and cross-section (five regions of Canada) information. Griffin and Gregory estimate a three-inputs (K, L, E) translog cost function from pooled international data (four years: 1955, 1960, 1965, 1969) for the manufacturing sector of nine industrialized countries. Table 4 summarizes some comparable estimates from the five studies.

TABLE 4

COMPARISON OF FIVE STUDIES ON ENERGY

			τ_{EE}	τ_{LL}	τ_{KK}	σ_{EL}	σ_{EK}	σ_{LK}
Berndt/Wood	USA 1947–71		−.47	−.45	−.48	.65	−3.22	1.01
Hudson/Jorgenson[20]	USA 1947–71		.07	−.45	−.42	2.16	−1.39	1.09
Fuss	Ontario (Can.) 1961–71		−.49	−.49	−.76	2.41	−.02	.86
Magnus[21]	Netherlands 1950–76	GCD. 1	−.16	−.32	−.48	1.25	−2.32	.89
		GCD. 2	−.29	−.19	−.37	.95	−2.08	.66
		TL. 1	−.20	−.30	−.46	1.19	−2.06	.85
		TL. 2	−.57	−.17	−.42	.86	−.30	.60
Griffin/Gregory	USA 1965		−.79	−.12	−.18	.87	1.07	.06
	Netherlands 1965		−.78	−.26	−.38	.86	1.02	.41

We conclude that our findings are in general agreement with the past literature. For our first model (based on π^*) the GCD and translog function yield very similar results. The second model, however, seems more sensitive to the choice of functional form. The most striking difference among the five studies is that Griffin and Gregory find E and K to be substitutes rather than complements.

[19] See Diewert [1974].

[20] Estimates taken from Griffin and Gregory [1976].

[21] Midpoint sample year 1963.

They argue that observations among countries will tend to reflect long-run adjustments, whereas time-series data will reflect short-run adjustments. Thus, they conclude that energy and capital, though complements in the short run, will be substitutes in the long run.

7. CONCLUDING REMARKS

This paper has sought to estimate factor demand relations in a three factor demand model that allows for considerable freedom in the variation of the substitution parameters. One of the major problems has been to collect adequate data on factor prices. Especially the price index of capital services is by no means ideal and requires improvement.[22] Other shortcomings in our analysis are that the GCD model requires predetermined factor prices and instantaneous adjustment in respect to all factors of production. Besides, it is found that in some years the substitution matrix is not negative semidefinite, thus violating the underlying theory of the model.

Nevertheless the study finds evidence that energy interacts with labor (substitutable) and capital (complementary) in different ways. This justifies the inclusion of energy as a separate input in the production function. The results can be used to assess the effect of energy price changes on energy use and total output. To carry out these projections in the context of our model we must predict future factor prices and apply the appropriate formulae. We then find predictions of the cost shares. From these we can derive projections of energy consumption, labor quantity and the volume of capital stock, if we have projections of output z.[23]

The complementarity between energy and capital has two interesting implications for investment policy: First, higher priced energy will — ceteris paribus — dampen the demand for new plant and equipment[24] and, to the extent that productivity gains are embodied in new plant and equipment, this may slow down the rate of productivity growth. Secondly, investment incentives such as accelerated depreciation allowances and investment tax credits result in increased demand for energy. This calls for a cautious use of these instruments.

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[22] One way out is to study only marginal effects. We can then use the price of investment goods. Such an approach would, however, require a completely new model.

[23] Also sufficient are projections of the costs or quantities of any of the factors.

[24] But it will stimulate employment, as $\sigma_{EL} > 0$.

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